Math 70 13.1 — Lessons 48 & 49

Chapter 1 Overview Conic Sections

13.1 Parabola and Circle
13.2 Ellipse and Hyperbola 13.3
13.4 Solving nonlinear systems of equations
13.5 Graphing systems of inequalities

In , we will have 4 types of equations, for which we need to

- notice algebraic features that are important
- identify shape that goes with these equations

Parabola: has one squared variable — x^2 and y OR y^2 and x

Circle:
- has two squared variables x^2 and y^2
- squared variables added $x^2 + y^2$
- $ax^2 + ay^2$ same coefficient on x^2 as on y^2

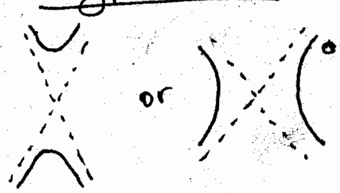
Ellipse:
- same as circle, except
- $ax^2 + by^2$ different coefficient on x^2 vs. y^2.

Hyperbola
- has two squared variables also; x^2 & y^2
- squared variables subtracted

$ax^2 - by^2$

$ay^2 - bx^2$

- coefficients irrelevant — can be same or different.



These four shapes are called conics or conic sections because they occur by slicing a double cone in different ways. To see a picture use "read textbook" link.

GC 34 – using GC to help graph sideways parabolas
GC 35 – ZOOM SQUARE to graph circles on GC

Math 70: Parabola and Circle

Objectives:

1) Given two points, calculate the coordinates of the midpoint using the midpoint formula $M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$. [Average the *x*-coordinates, average the *y*-coordinates.]
2) Given two points, calculate the distance between them using the distance formula $D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ [Pythagorean theorem, solve for *c*.]
3) Graph parabolas
   a. Study the equation's structure first, determine if it's up/down (chap 8) or left/right (chap 10)!
   b. Review up/down parabolas which are functions $y=a(x-h)^2+k$ or $y=ax^2+bx+c$
      - Vertex (h,k) or $h=-\frac{b}{a}$ and $k=y(h)$
      - $a>0$ opens up, $a<0$ opens down
      - $|a|=1$ standard shape, $|a|>1$ narrower than standard, $|a|<1$ wider than standard
      - Equation of the axis of symmetry $x=h$
   c. Graph parabolas that open left or right and are not functions $x=a(y-k)^2+h$
      - Swap *x* and *y*!!!
      - Vertex (h,k) with *h* outside the parentheses, and *k* inside the parentheses (reversed!)
      - Vertex formula $k=-\frac{b}{a}$ (*y*-coordinate!) and $h=x(h)$
      - $a>0$ opens right, $a<0$ opens left
      - $|a|=1$ standard shape, $|a|>1$ narrower than standard, $|a|<1$ wider than standard
      - Equation of the axis of symmetry $y=k$ (horizontal! Use the *y*-coordinate of vertex.)
4) Graph circles from equations in standard form $(x-h)^2+(y-k)^2=r^2$, where the center is (h,k) and the radius is *r*.
5) Re-write the general form of an equation of a circle $ax^2+by^2+cx+dy+e=0$ in standard form using completing the square, twice!
6) Given the center and radius, write the equation of a circle in standard form.

Examples and Practice

1) Calculate the midpoint between $\left(-4,\frac{19}{3}\right)$ and $\left(3,-\frac{13}{3}\right)$

2) Calculate the distance between $\left(-4,\frac{19}{3}\right)$ and $\left(3,-\frac{13}{3}\right)$

3) Find the vertex, direction, axis of symmetry of $y=-\frac{1}{2}(x-3)^2-4$

4) Sketch graph of $x=y^2$

5) Sketch graph of $x=-\frac{1}{2}(y-1)^2-4$

6) Use $x=2y^2+4y+5$.
   a. Find the vertex.
   b. Rewrite the equation in the form $x=a(y-k)^2+h$

7) Sketch graph of $(x+3)^2+(y-1)^2=25$

8) Sketch graph of $(x-1)^2+y^2=9$

9) Find the center and radius of $x^2+y^2+4x-8y-16=0$

10) Find the center and radius of $2x^2+2y^2-\frac{1}{2}=0$

11) Write the equation of a circle (in standard form) having center $(-7,3)$ and radius $\frac{2}{3}$.

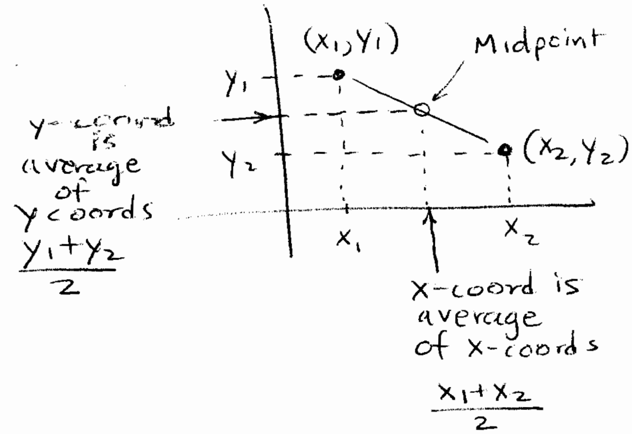
12) Use $x=y^2+6y+2$
   a. Find the x-intercept
   b. Find the y-intercept.

Review:

Midpoint Formula

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

coordinates of a point that's halfway between (x_1,y_1) and (x_2,y_2)



Calculate the midpoint between

SKIP ① $(2,3)$ and $(6,7)$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$=\left(\frac{2+6}{2}, \frac{3+7}{2}\right)$$

$$=\left(\frac{8}{2}, \frac{10}{2}\right)$$

$$=\boxed{(4,5)}$$

Yes ② $\left(-4, \frac{19}{3}\right)$ and $\left(3, \frac{-13}{3}\right)$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$=\left(\frac{-4+3}{2}, \frac{19/3+{-13}/3}{2}\right)$$

$$=\left(\frac{-1}{2}, \frac{6/3}{2}\right)$$

$$=\left(-\frac{1}{2}, \frac{2}{2}\right)$$

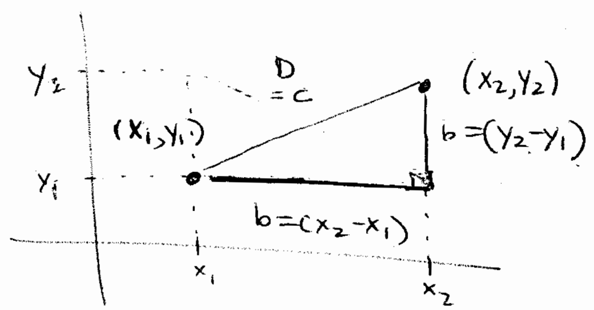
$$=\boxed{\left(-\tfrac{1}{2}, 1\right)}$$

Review:

Distance Formula

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

length of a line segment connecting (x_1,y_1) and (x_2,y_2)



$a^2+b^2=c^2$ Pythagorean Theorem

$$\sqrt{a^2+b^2}=c$$

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=D$$

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Calculate the distance between the points.

SKIP ③ (2, 3) and (5, 7)

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$D = \sqrt{(2-5)^2 + (3-7)^2}$

$D = \sqrt{(-3)^2 + (-4)^2}$

$D = \sqrt{9 +}$

$D = \sqrt{25}$

$\boxed{D = 5}$

YES ④ $(-4, \frac{19}{3})$ and $(3, \frac{-13}{3})$

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$D = \sqrt{(3 - -4)^2 + (\frac{-13}{3} - \frac{19}{3})^2}$

$D = \sqrt{7^2 + (\frac{32}{3})^2}$

$D = \sqrt{49 + \frac{1024}{9}}$

$D = \sqrt{\frac{441}{9} + \frac{1024}{9}}$

$D = \sqrt{\frac{1465}{9}}$

$D = \frac{\sqrt{1465}}{\sqrt{9}}$

$D = \boxed{\frac{\sqrt{1465}}{3}}$

1465 → 5, 293
can't simplify

Remember: Graphing up/down parabolas (chapter 8)

$y = a(x-h)^2 + k$

(h, k) = vertex

vertex formula
$h = \frac{-b}{2a}$ k = evaluate at $x = h$

$a > 0$ opens up, $a < 0$ opens down

(a positive, positive y-direction) (a negative, negative y-direction)

$|a| > 1$ narrower than $y = x^2$

$|a| < 1$ wider than $y = x^2$

$x = h$ equation of axis of symmetry
(vertical line through vertex x coord)

YES ④ Sketch $y = -\frac{1}{2}(x-3)^2 - 4$

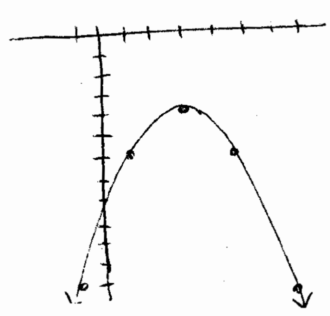
vertex $(3, -4)$

opens down, wide

right 1 down $-\frac{1}{2}(1)^2 = -\frac{1}{2}$

right 2 down $-\frac{1}{2}(2)^2 = -2$

right 4 down $-\frac{1}{2}(4)^2 = -8$



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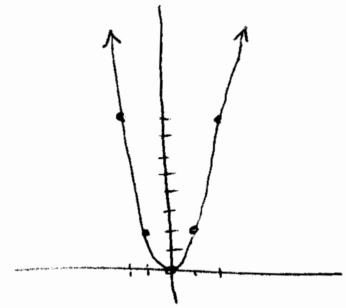
Extra: [SKIP] (5) Sketch $y = 2x^2$

vertex (0,0)
opens up
narrower

right 1 up $2(1)^2 = 2$
right 2 up $2(2)^2 = 8$
right 3 up $2(3)^2 = 18$



GRAPHING LEFT/RIGHT PARABOLAS

In the equation x and y trade places

$$x = a(y-k)^2 + h$$

CAUTION (h,k) is vertex, but h is outside ( )
k is inside next to y.

$a > 0$ opens right — a positive; positive x-direction

$a < 0$ opens left — a negative; negative x-direction

$|a| > 1$ narrower (unchanged)
$|a| < 1$ wider (unchanged)

$y = k$ axis of symmetry is horizontal
(still passes through vertex, but using y-coordinate)

** CAUTION ** vertex formula finds y-coordinate

$$y = k = \frac{-b}{2a}$$ ($x = h$ evaluate at $y = k$)

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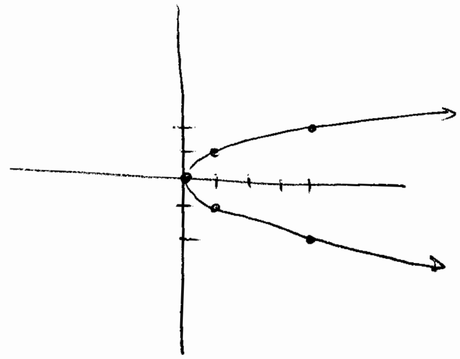
Sketch graphs

YES (6) $x = y^2$

$a = 1$, opens right

vertex $(0,0)$

axis of sym $y = 0$



{ down 1 right $1^2 = 1$
{ up 1 right $1^2 = 1$

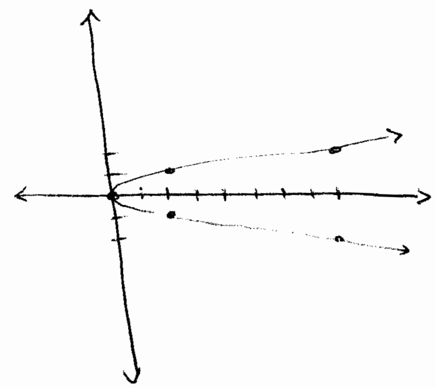
{ down 2 right $2^2 = 4$
{ up 2 right $2^2 = 4$

SKIP (7) $x = 2y^2$

vertex $(0,0)$

axis of sym $y = 0$

$a = 2$ narrower, opens right



{ down 1 right $2(1)^2 = 2$
{ up 1 right $2(1)^2 = 2$

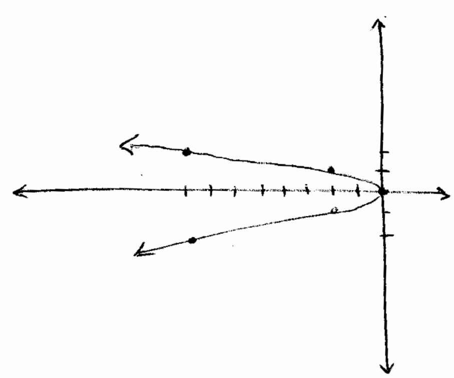
{ down 2 right $2(2)^2 = 8$
{ up 2 right $2(2)^2 = 8$

SKIP (8) $x = -2y^2$

vertex $(0,0)$

axis of sym $y = 0$

$a = -2$ narrower, opens left



{ down 1 left $-2(1)^2 = -2$
{ up 1 left $-2(1)^2 = -2$

{ down 2 left $-2(2)^2 = -8$
{ up 2 left $-2(2)^2 = -8$

Once you know the direction and how much to multiply your pattern of squares, it's often easier to just turn your paper (rather than get up/down/left/right confused).

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Sketch graphs

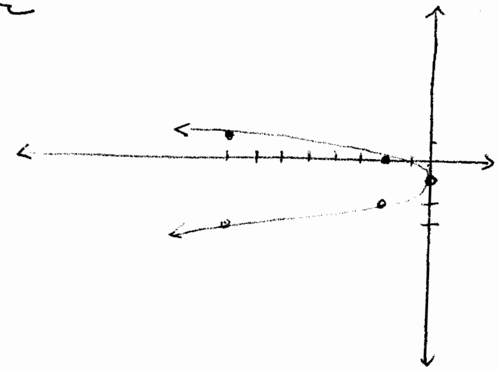
(9) $x = -2(y+1)^2$

SKIP

vertex $(0,-1)$ y-coord is next to y-variable!

axis of sym $y = -1$

$a = -2$ opens left, narrower



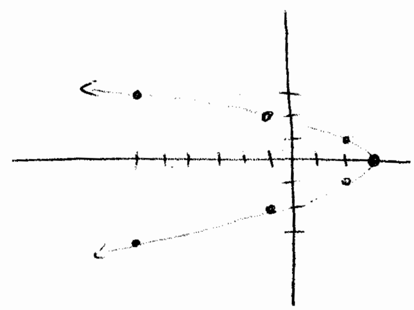
(10) $x = -y^2 + 3$

SKIP

vertex $(3, 0)$

axis of sym $y = 0$

$a = -1$ opens left, basic shape

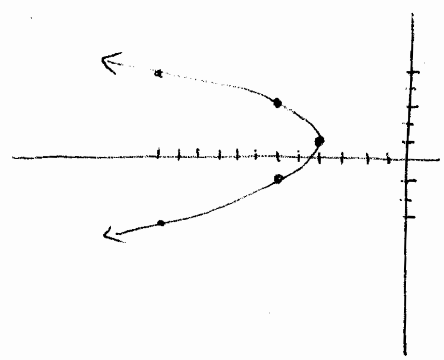


YES (11) $x = -\frac{1}{2}(y-1)^2 - 4$

vertex $(-4, 1)$

axis of sym $y = 1$

$a = -\frac{1}{2}$ opens left, narrower



down 1 left $-\frac{1}{2}(1)^2 = -\frac{1}{2}$ yuck

down 2 left $-\frac{1}{2}(2)^2 = -2$ yes

down 4 left $-\frac{1}{2}(4)^2 = -8$ yes

Study equation's structure first, to determine if its
left/right chapter 10
up/down chapter 8

vertex is most important point, and wrong if you have direction wrong

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Write equations given as $x = ay^2 + by + c$ in the form $x = a(y-k)^2 + h$.

(12) Review: write $y = -x^2 - 2x + 15$ in the form $y = a(x-h)^2 + k$.

SKIP

$y = -(x^2 + 2x \qquad) + 15$ — factor out a from x^2 and x terms

$\# = \frac{2}{2} = 1$

$\#^2 = 1^2 = 1$ — find CTS #, $\#^2$

$y = -(x^2 + 2x + 1) + 15 - (-1)(1)$

add CTS # inside; subtract a · CTS # outside (a, CTS#)

$$\boxed{y = -(x+1)^2 + 16}$$

write perfect square using CTS #

(13) $x = 2y^2 + 4y + 5$

SKIP

$x = 2(y^2 + 2y \qquad) + 5$

$\# = \frac{2}{2} = 1$

$\#^2 = 1^2 = 1$

$x = 2(y^2 + 2y + 1) + 5 - 2(1)$

$$\boxed{x = 2(y+1)^2 + 3}$$

(14) $x = -y^2 - 4y - 1$

SKIP

$x = -(y^2 + 4y + 4) - 1 - (-1)(4)$

$\# = \frac{4}{2} = 2$

$\#^2 = 2^2 = 4$

$$\boxed{x = -(y+2)^2 + 3}$$

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Sketch graphs.

(15) $x = 2y^2 + 4y + 5$

YES

find vertex by vertex formula or CTS.

$$y = \frac{-b}{2a}$$

$$y = \frac{-4}{2(2)} = -1$$

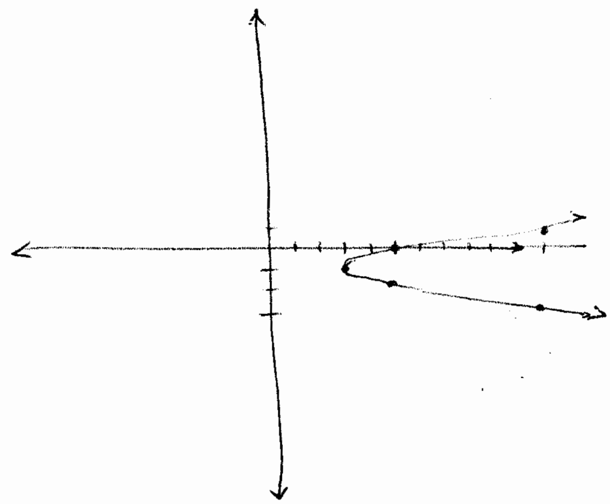
$$x = 2(-1)^2 + 4(-1) + 5$$
$$= 2 - 4 + 5$$
$$= 3$$

vertex $(3, -1)$

[Note: This matches CTS work in (13).]

$a = 2 > 0$ narrower

opens right



Note: You can check left/right graphs in GC by reversing roles of x and y

$y_1 = 2x^2 + 4x + 5$

TABLE

x	y
-1	3
0	5
1	11
-2	5
-3	11

on graph $y = 2x^2 + 4x + 5$

MEANS

x	y
3	-1
5	0
11	1
5	-2
11	-3

on graph $x = 2y^2 + 4y + 5$

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Sketch graphs

(16) $x = -y^2 - 4y - 1$

SKIP

find vertex · by vertex formula

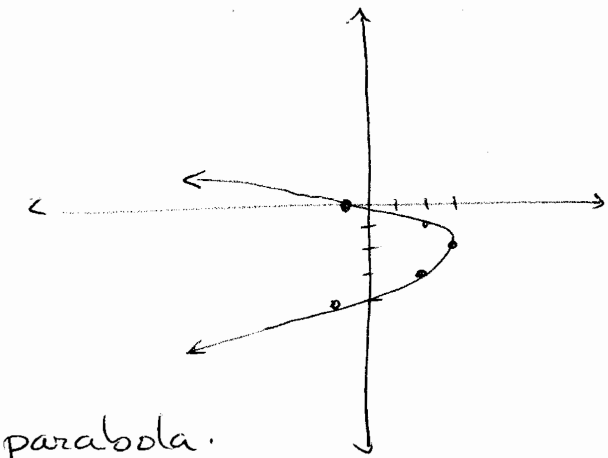
$$y = -\frac{b}{2a} = -\frac{(-4)}{2(-1)} = \frac{4}{-2} = -2$$

$$x = -(-2)^2 - 4(-2) - 1$$
$$= -4 + 8 - 1$$
$$= 3$$

vertex $(3, -2)$

[Note: this matches CTS work in (14).]

$a = -1$ basic shape opens left



(17) $y = -x^2 - 2x + 15$

SKIP

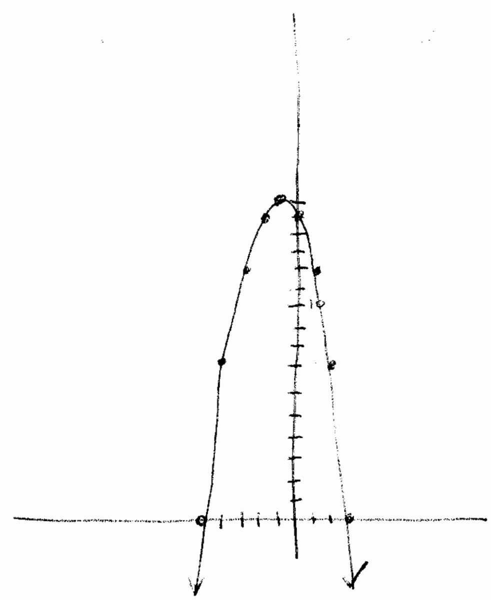
* Notice x's and y's! This is an up/down chap 8 parabola.

vertex $x = -\frac{b}{2a} = \frac{-(-2)}{2(-1)} = \frac{2}{-2} = -1$

$$y = -(-1)^2 - 2(-1) + 15$$
$$= -1 + 2 + 15$$
$$= 16$$

vertex $(-1, 16)$

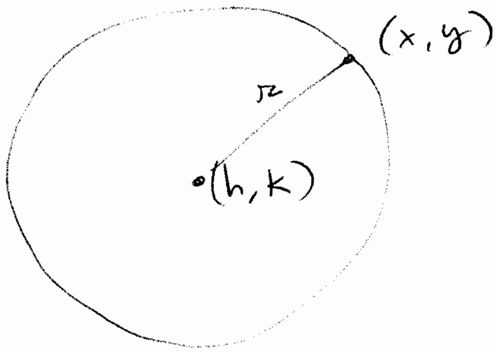
$a = -1$ opens down basic shape



Math 70 MG 5/e 10.1 Circles

Objectives
1) Graph circles from equations in standard form $(x-h)^2 + (y-k)^2 = r^2$
2) Write equations of circles in standard form when given as
$$ax^2 + bx + cy^2 + dy = e$$
3) Write equations of circles in standard form given center (h, k) and radius r.

A circle is a set of all points (x, y) that are the same distance (called the radius) from a single point (called the center (h,k))



distance formula
$$r = \sqrt{(x-h)^2 + (y-k)^2}$$
square both sides
$$r^2 = (x-h)^2 + (y-k)^2$$
change sides
$$(x-h)^2 + (y-k)^2 = r^2$$

This is called the <u>standard form</u> of the equation of a circle because we can determine the center and radius by looking at the equation.

Graph.

yes ① $(x+3)^2 + (y-1)^2 = 25$

$(x-h)=0$	$y-k=0$	r^2	<u>Step 1</u>: Find coordinates of center and value of radius.
$x=h$	$y=k$	$\sqrt{r^2}=r$	
↑	↑	↑	
$x+3=0$	$y-1=0$	$\sqrt{25}=r$	
$x=-3$	$y=1$	$r=5$	
x coord of center	y coord of center		

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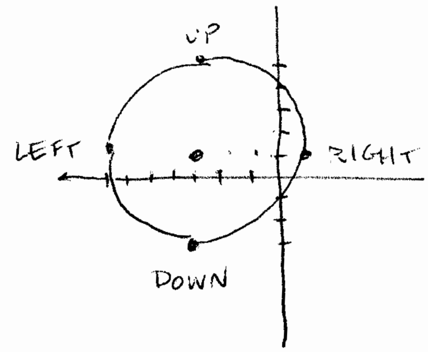
① can't graph.

<u>step 2</u>: plot center

(-3, 1)



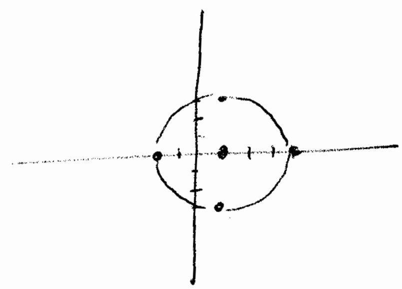
<u>step 3</u>: use radius to plot four points left, right, up, and down from center.

<u>step 4</u>: Connect the four dots with smooth round curves to make a circle.

[yes] ② $(x-1)^2 + y^2 = 9$

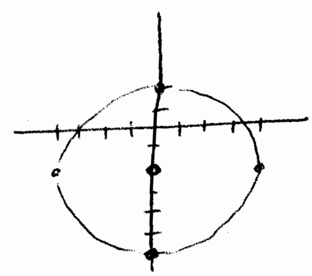
$x = 1$ $y = 0$ $r = \sqrt{9} = 3$

$(1, 0)$



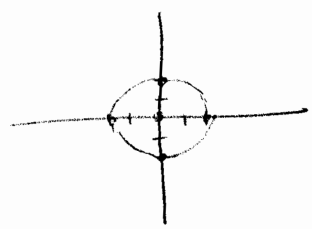
[SKIP] ③ $x^2 + (y+2)^2 = 16$

$x = 0$ $y = -2$ $r = \sqrt{16} = 4$



[SKIP] ④ $x^2 + y^2 = 4$

$x = 0$ $y = 0$ $r = \sqrt{4} = 2$



<u>Note</u>: Circles can be graphed on GC if you solve the equation for y:

$$y^2 = 4 - x^2$$

$$y = \pm\sqrt{4 - x^2}$$ ← 2 functions

$y_1 = \sqrt{(4 - x^2)}$

$y_2 = -\sqrt{(4 - x^2)}$

ZOOM SQUARE (5) makes it rounder

So it's not difficult to graph a circle if you know the center and radius. But how do we find them if we don't?

Write each equation in standard form and find the center and radius.

YES (5) $x^2 + y^2 + 4x - 8y = 16$

step 1: Collect x^2 and x together
y^2 and y together
and constants on RHS.

$$x^2 + 4x \quad + y^2 - 8y \quad = 16$$

step 2: Complete the square in x AND complete the square in y.

for x	for y
$\# = \frac{4}{2} = 2$	$\# = \frac{-8}{2} = -4$
$\#^2 = 2^2 = 4$	$\#^2 = (-4)^2 = 16$

* Be sure to add both CTS $\#^2$ to the RHS *

$$x^2 + 4x + 4 \quad + y^2 - 8y + 16 = 16 + 4 + 16$$

$$(x+2)^2 + (y-4)^2 = 36$$

center $(-2, 4)$
radius 6

YES (6) $2x^2 + 2y^2 = \frac{1}{2}$

divide all terms by 2

$$x^2 + y^2 = \frac{1}{4}$$

$$(x-0)^2 + (y-0)^2 = \left(\frac{1}{2}\right)^2$$

center $(0,0)$
radius $\frac{1}{2}$

Given the center and radius, write equation of the circle in standard form.

SKIP (7) center $(-7,3)$ radius 10.

step 1: write the standard form as a formula

$$(x-h)^2+(y-k)^2=r^2$$

step 2: substitute and simplify r^2

$(x-h)^2$ and $(y-k)^2$ can remain unsimplified.

$$(x-(-7))^2+(y-3)^2=10^2$$

$$\boxed{(x+7)^2+(y-3)^2=100}$$

SKIP (8) center $(0,-2)$ radius $\frac{2}{3}$

$$(x-0)^2+(y+2)^2=\left(\frac{2}{3}\right)^2$$

$$\boxed{x^2+(y+2)^2=\frac{4}{9}}$$

might also clear fractions:

$$\boxed{9x^2+9(y+2)^2=4}$$

combined version

YES (7+8) center $(-7,3)$ and radius $\frac{2}{3}$

$$(x+7)^2+(y-3)^2+\left(\frac{2}{3}\right)^2$$

$$\boxed{(x+7)^2+(y-3)^2=\frac{4}{9}}$$

or

$$\boxed{9(x+7)^2+9(y-3)^2=4}$$

Extras: Write in standard form and graph.

SKIP (9) $x^2+y^2+6x-2y=6$

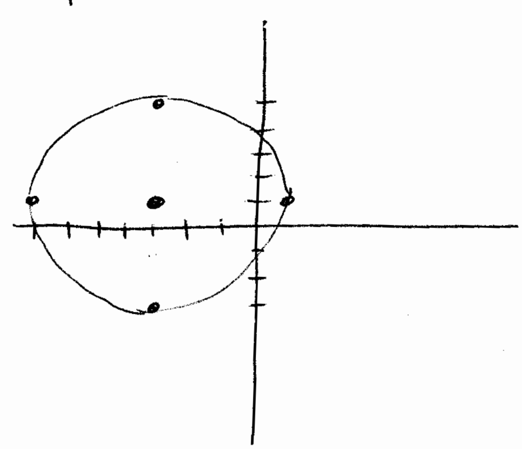
$$x^2+6x+9+y^2-2y+1=6+9+1$$

$$\left(\frac{6}{2}\right)^2=3^2=9 \qquad \left(-\frac{2}{2}\right)^2=(-1)^2=1$$

$$(x+3)^2+(y-1)^2=16$$

center $(-3,1)$

radius $\sqrt{16}=4$



Same graph as (1)

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SKIP (10) $x^2 + y^2 + 6x + 10y - 2 = 0$

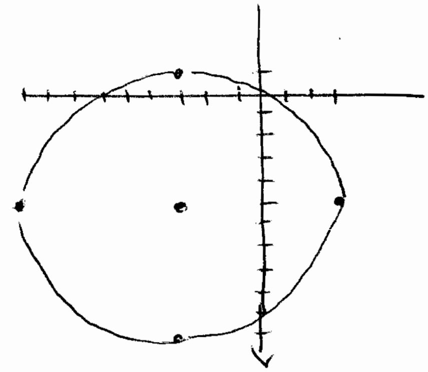
$x^2 + 6x + 9 + y^2 + 10y + 25 = 2 + 9 + 25$

$\left(\frac{6}{2}\right)^2 = 3^2 = 9$ $\left(\frac{10}{2}\right)^2 = 5^2 = 25$

$(x+3)^2 + (y+5)^2 = 36$

center $(-3, -5)$

radius $\sqrt{36} = 6$



SKIP (11) $x^2 + y^2 - 8y + 5 = 0$

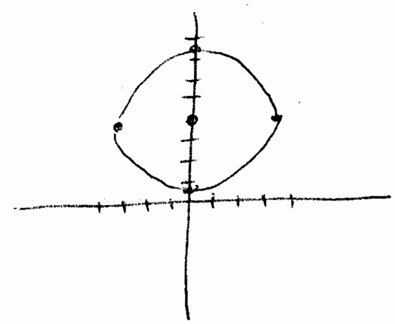
$x^2 + y^2 - 8y + 16 = -5 + 16$

$\uparrow$ $\left(\frac{0}{2}\right)^2 = 0$ $\uparrow$ $\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

$(x-0)^2 + (y-4)^2 = 11$

center $(0, 4)$

radius $\sqrt{11} \approx 3.3$



you won't be asked to graph $r = \sqrt{11}$!

Math 70 MG 5/e 10.1

Graph.

(12) $x = -2y^2 - 4y$

SKIP

Notice: No x^2! This is not a circle.

It's a parabola going left or right.

$x = -2(y^2 + 2y + 1) - (-2)(1)$

$\#^2 = \left(\frac{2}{2}\right)^2 = 1^2 = 1$

$x = -2(y+1)^2 + 2$

vertex $(2, -1)$
opens left
narrow

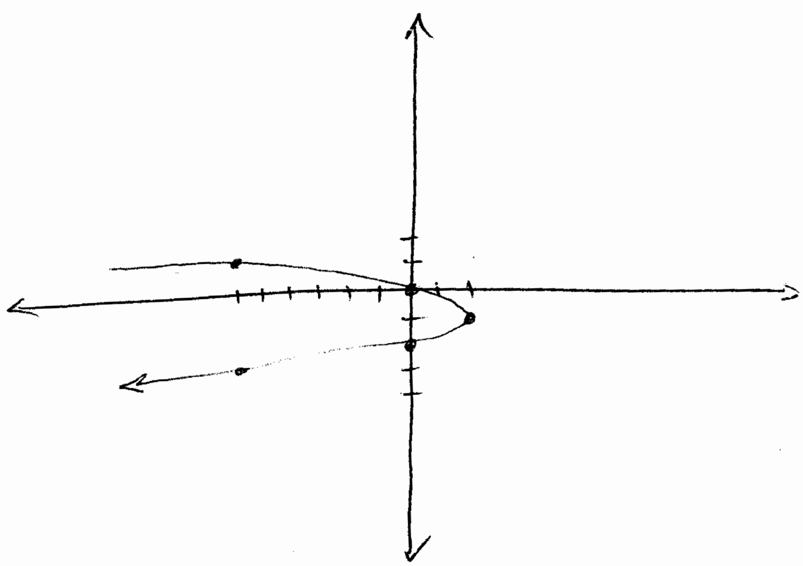
-OR-

vertex formula

$y = \frac{-b}{2a} = \frac{-(-4)}{2(-2)} = \frac{4}{-4} = -1$

$x = -2(-1)^2 - 4(-1)$
$= -2 + 4 = 2$

vertex $(2, -1)$
$a = -2$ opens left
narrow.



Math 70 MG 5/e 10.1

Graph.

(13) $x = y^2 + 6y + 2$

YES

no x^2! Again a parabola

x & $y^2 \Rightarrow$ left or right

$x = (y^2 + 6y + 9) + 2 - 9$

$\left(\frac{6}{2}\right)^2 = 3^2 = 9$

$x = (y+3)^2 - 7$

vertex $(-7, -3)$ $a = 1$ opens right

- OR -

vertex formula

$y = \frac{-b}{2a} = \frac{-6}{2(1)} = -3$

$x = (-3)^2 + 6(-3) + 2$

$= 9 - 18 + 2$

$= -7$

vertex $(-7, -3)$

$a = 1$ opens right

(14) Find the y-intercepts in (13).

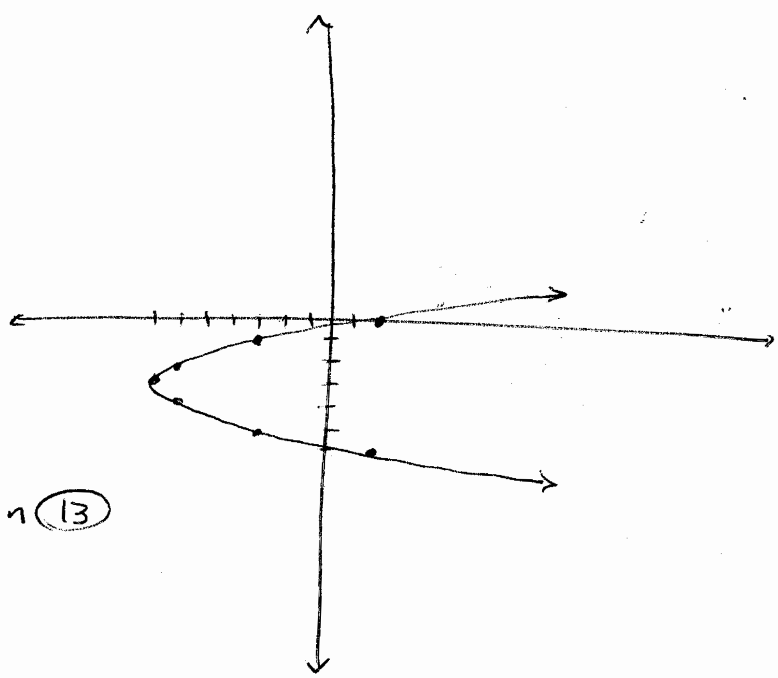
set $x = 0$ solve for y.

$0 = (y+3)^2 - 7$

$7 = (y+3)^2$

$\pm\sqrt{7} = y + 3$

$\boxed{-3 \pm \sqrt{7} = y}$



(15) Find the x-intercept in (13)

Set $y = 0$ solve for x.

$x = 0^2 + 6(0) + 2$

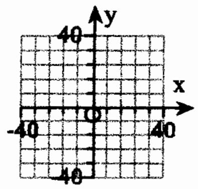
$\boxed{x = 2}$

10.1.59 Sketch the graph of the equation. If the graph is a parabola, find its vertex. If the graph is a circle, find its center and radius.

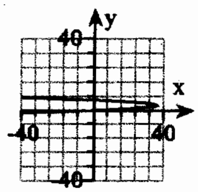
$x = -4y^2 + 24y$

Choose the correct graph below.

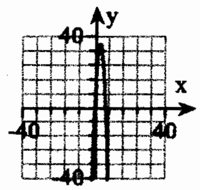
○ A.



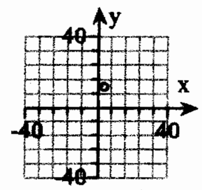
✔ B.



○ C.



○ D.



Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

✔ A. The graph is a parabola with the vertex located at [redacted].
(Type an ordered pair.)

○ B. The graph is a circle with radius [redacted].
The center is located at [redacted].
(Type an ordered pair.)

CTS

$x = -4(y^2 - 6y + 9) + 36$

-36

$x = -4(y-3)^2 + 36$

vertex (36, 3)

left because x, y^2 and $a = -4 < 0$.

1

Rev 8-8-13

Name ______________________________

Date ______________________________

TI-84+ GC 34 Using GC to Graph Parabolae that are Not Functions of x

Objectives: Recall the square root property
Practice solving a quadratic equation for y
Graph the two parts of a horizontal parabola
Identify the axis of symmetry and its equation

When solving for a variable or expression that has been squared, recall that there are usually two solutions. Example: $x^2 = 9$ can be factored to $(x+3)(x-3) = 0$, giving solutions $x = 3, -3$.
The square root property tells us to use $\pm$ when we take the square root of both sides of $x^2 = 9$.

Solve each of the following equations for y by isolating the square and using the square root property.

1) $x = y^2$

2) $x = (y-2)^2$

3) $x = 5(y-2)^2$

4) $x = 5(y-2)^2 - 4$

5) $x = -5(y-2)^2 - 4$

When we graph using the GC, we have only the Y= menu, not an X= menu.
So we must have y as a function of x, not y^2 = an expression in x, or x = an expression in y.
Solve for y first, using algebra. If the square root property was used, write each of the two functions.

6) Graph $x = y^2$ on the GC.

Step 1: Solve the equation for y using the square root property;
$\pm\sqrt{x} = \sqrt{y^2}$ gives $y = \pm\sqrt{x}$.

Step 2: Write as two functions.
$y_1 = \sqrt{x}$ and $y_2 = -\sqrt{x}$

Step 3: Enter these two functions into the GC Y= menu and graph.

[Y=] [CLEAR] [2nd] [x^2] [X,T,θ,n] [)] [ENTER]

[CLEAR] [(−)] [2nd] [x^2] [X,T,θ,n] [)] [ENTER] [ZOOM] [6]

The vertex of a horizontal parabola can be found from the standard form of the equation, just as it was for a quadratic function. The roles of x and y are reversed.
$x = a(y-k)^2 + h$ is the standard form of the equation, and (h,k) is the vertex.
The axis of symmetry is a horizontal line through the vertex, with equation $y = k$.
Notice that the y-coefficient of the vertex, k, is inside the parentheses, next to the y variable.
If a>0, the parabola opens in the positive x-direction, to the right.
If a<0, the parabola opens in the negative x-direction, to the left.

7) Find the vertex of $x = (y-2)^2$ Does this parabola open left or right?

8) Find the equation of the axis of symmetry.

9) Solve $x = (y-2)^2$ for its two functions. Make a table on the GC for the two functions.

Step 1: Solve the equation for y using the square root property; (Use your previous work.)

Step 2: Write as two functions.

Step 3: Enter these two functions into the GC Y= menu and graph.

[Y=] [CLEAR] [2] [+] [2nd] [x^2] [X,T,Θ,n] [)] [ENTER] [CLEAR] [2] [−]
[2nd] [x^2] [X,T,Θ,n] [)] [ENTER]

Step 4: Set up the GC table to begin at the vertex.

If the parabola opens left, you'll use the [▲] key to move through the table.

If the parabola opens right, use [▼] to move through the table.

[2nd] [WINDOW] [0] [2nd] [GRAPH]

Use the GC table to fill in this chart:

x-value	$y_1 = 2 + \sqrt{x}$	$y_2 = 2 - \sqrt{x}$
0		
1		
4		
9		

10) Notice that the x-values provided to you in the previous chart have a pattern. What is it?

11) Find the vertex of $x = 5(y-2)^2$ Does this parabola open left or right?

12) Find the equation of the axis of symmetry of $x = 5(y-2)^2$.

13) Solve $x = 5(y-2)^2$ for its two functions. Make a table on the GC for the two functions.

Step 1: Solve the equation for y using the square root property; (Use your previous work.)

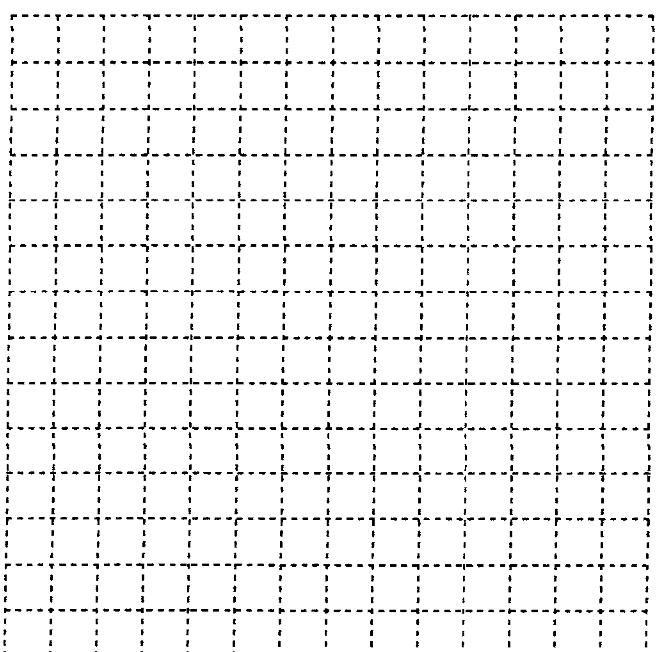
Step 2: Write as two functions.

Step 3: Enter these two functions into the GC Y= menu and graph.

Step 4: Set up the GC table to begin at the vertex.
Use the GC table to fill in this chart:

x-value	$y_1 =$	$y_2 =$
0		
5		
20		
45		

14) Draw axes and label them with an appropriate scale. Plot the points and graph both of the curves, to get the horizontal parabola. Sketch the axis of symmetry.



Graph. You may wish to use these steps:

a. Does this parabola open left or right?
b. What is the vertex?
c. What is the equation of the axis of symmetry?
d. Solve for y, write two functions.
e. Make a table.
f. Draw axes and label them with an appropriate scale. Plot the points and graph parabola. Sketch the axis of symmetry.

15) Graph $x = 5(y-2)^2 - 4$

Shortcut: The ordered pairs (x,y) that satisfy $y = -5(x-2)^2 - 4$ can be changed (swap x for y and y for x) to get ordered pairs that satisfy $x = -5(y-2)^2 - 4$. Because $y = -5(x-2)^2 - 4$ is already a function of x, no solving for y is needed. To use this shortcut, skip step d. above, and make a table of values for $y = -5(x-2)^2 - 4$. Then make a second table for graphing $x = -5(y-2)^2 - 4$ by swapping x for y and y for x. If this shortcut doesn't make sense to you, don't use it.

TI-84+ GC 34 Using GC to Graph Parabolae that are Not Functions of x, page 5

Graph. You may wish to use these steps:

a. Does this parabola open left or right?
b. What is the vertex?
c. What is the equation of the axis of symmetry?
d. Solve for y, write two functions. (Or use the shortcut)
e. Make a table.
f. Draw axes and label them with an appropriate scale. Plot the points and graph parabola. Sketch the axis of symmetry.

16) Graph $x = -5(y-2)^2 - 4$

TI-84+ GC 34 Using GC to Graph Parabolae that are Not Functions of x, solutions, page 6

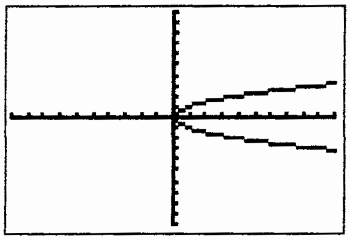
1) $y = \pm\sqrt{x}$

2) $y = 2 \pm \sqrt{x}$

3) $y = 2 \pm \sqrt{\frac{x}{5}}$ or $y = 2 \pm \frac{\sqrt{5x}}{5}$

4) $y = 2 \pm \sqrt{\frac{x+4}{5}}$ or $y = 2 \pm \frac{\sqrt{5(x+4)}}{5}$

5) $y = 2 \pm \sqrt{\frac{x+4}{-5}}$ or $y = 2 \pm \sqrt{\frac{-x-4}{5}}$ or $y = 2 \pm \frac{\sqrt{-5(x+4)}}{5}$



6)

7) V(0,2) a=1>0, opens right.

8) $y = 2$

```
TABLE SETUP
 TblStart=0
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

9) $y = 2 \pm \sqrt{x}$, $y_1 = 2 + \sqrt{x}$ $y_2 = 2 - \sqrt{x}$ cont

X	Y1	Y2
0	2	2
1	3	1
2	3.4142	.58579
3	3.7321	.26795
4	4	0
5	4.2361	-.2361
6	4.4495	-.4495

X=0

x-value	$y_1 = 2 + \sqrt{x}$	$y_2 = 2 - \sqrt{x}$
0	2	2
1	3	1
4	4	0
9	5	-1

10) The x-values are all perfect squares.

11) V(0,2)

12) $y = 2$

13) $y = 2 \pm \sqrt{\frac{x}{5}}$ or $y = 2 \pm \frac{\sqrt{5x}}{5}$; $y_1 = 2 + \sqrt{\frac{x}{5}}$, $y_2 = 2 - \sqrt{\frac{x}{5}}$

Rev 8-8-13

Name ______________________________

Date ______________________________

TI-84+ GC 35 ZOOM Square for Circles

Objectives: Graph several semi-circles simultaneously on the GC
Use the VARS menu to refer to another y in the Y= menu and save typing
Use ZOOM Square to make the circles round on the GC screen
Notice the limitation of GC graphs for circles

> The VARS button has two menus: VARS and Y-VARS. Y-VARS refers the Y= menu.
> If we have defined y_1 in the Y= menu, and y_2 is the opposite of y_1, we can use the Y-VARS menu to make $y_2 = -y_1$

1) What shape is the graph of $x^2 + y^2 = 25$?

2) Solve $x^2 + y^2 = 25$ for y, put these two equations into GC as y_1 and y_2.

[Y=] [CLEAR] [2nd] [x^2] [2] [5] [−] [X,T,θ,n] [x^2] [)]

Define y_2 as $y_2 = -y_1$

[(−)] Open VARS: [VARS]. Move to Y-VARS: [▶]. Select Function: [ENTER]. Select y_1: [ENTER]. Move to next: [ENTER].

3) What shape is the graph of $y = -\sqrt{6.25 - x^2}$?

4) Put $y = -\sqrt{6.25 - x^2}$ as y_3 in GC.

[(−)] [2nd] [x^2] [6] [.] [2] [5] [−] [X,T,θ,n] [x^2] [)] [ENTER]

5) What shape is the graph of $(x+2)^2 + (y-2)^2 = 1$?

6) Solve $(x+2)^2 + (y-2)^2 = 1$ for y; put these two equations into GC as y_4 and y_5

[2] [+] [2nd] [x^2] [1] [−] [(] [X,T,θ,n] [+] [2] [)] [x^2]
[)] [ENTER] [2] [−] [2nd] [x^2] [1] [−] [(] [X,T,θ,n] [+] [2]
[)] [x^2] [)] [ENTER]

TI-84+ GC 35 ZOOM Square for Circles page 2

7) What shape is the graph of $(x-2)^2+(y-2)^2=1$?

8) Solve $(x-2)^2+(y-2)^2=1$ for y; put these two equations into GC as y_6 and y_7

[2] [+] [2nd] [x^2] [1] [−] [(] [X,T,θ,n] [−] [2] [)] [x^2] [)] [ENTER]

[2] [−] [2nd] [x^2] [1] [−] [(] [X,T,θ,n] [−] [2] [)] [x^2] [)] [ENTER]

9) Graph in a standard window: [ZOOM] [6]. What errors does the GC make?

10) Graph in a square window: [ZOOM] [5]. Which error(s), if any, does this window fix?

11) Sketch the graph of all of these semicircles on the same axes. Use what you know about how the center and radius of a circle appear in its equation. Sketch the axes very lightly. You may wish to use a GC table from $x=-5$ to $x=5$. Be sure your graph corrects the GC errors.